Every Student Counts

Professional Development Guide
Component Two for
Elementary School Level

Number & Operations and Algebra

Teaching Addition and Subtraction Facts and Algorithms

Year 1 - Day 2

Teaching Addition and Subtraction Facts and Algorithms

Unit Overview

Activity	Description	Time	Materials
1.Assessing student understanding of number	• Participants learn how to assess understanding of number combinations through ten using The Hiding Assessment and how to assess the understanding of the operations of addition and subtraction using The Static Set Assessment	15 min.	Overhead 1; Handout 1; unifix cubes; Handout 2; Overhead 1b
2.A predominantly constructivist approach for teaching the addition and subtraction facts	• Participants experience the use of "meaningful flash cards" to promote understanding of number combinations and part-whole relationships • Participants practice story telling techniques using story boards to promote understanding of addition/subtraction operations	30 min.	Kathy Richardson videos, *Developing Number Concepts* by Kathy Richardson - Book 2; Meaningful flash cards; story boards; counters; Handout 3; Overhead 2
3.A more behaviorist approach for teaching the addition and subtraction basic facts	• Participants practice strategies for learning the addition/ subtraction facts with Cuisenaire Rods. • Sample activities, warm-ups, and daily practice from a commercial source for learning the facts are shown. • Participants discuss the role of timed fact tests	30 min.	Overhead 3; Handout 4; Cuisenaire Rods; *Fish Eyes*; *Anno's Counting House, 1 Hunter*; colored overhead pens; Handout 5a -5l; Larry Leutzinger's *Facts That Last*
4.A more behaviorist approach for teaching the addition and subtraction algorithms	• Participants practice using base ten blocks, place value mats, and numeral cards or playing cards for multiple-digit addition and subtraction • Participants learn the "Front End" method for estimating sums based on the base ten model	15 min.	Handout 6; base ten blocks, numeral cards, dice; base ten blocks for the overhead; Overhead 4; Overhead 5; Handout 7; Overhead 6; Overheads 7a-7b
5.A predominantly constructivist approach for teaching the addition and subtraction algorithms	• Participants view and discuss a video showing the difficulties children often have when asked to solve problems using procedures they don't understand and then showing children solving problems with understanding	35 min.	*Thinking with Numbers, Volume 2, Video 1* by Kathy Richardson

6. Computational Procedures	• Participants will examine computation issues.	115 Min	*Bridges Unit* "Computational Procedures" Handouts and materials

Summary of Session:

- In the first activity, participants learn how to conduct the Hiding Assessment and the Static Set Assessment.
- In the next activity, participants become familiar with a student-centered, constructivist approach to teaching the basic facts.
- In the third activity, participants practice strategies for learning the addition/subtraction facts with Cuisenaire Rods; sample activities, warm-ups, and daily practice from a commercial source for learning the facts will be shown, and participants will discuss the role of timed fact tests.
- In the fourth activity, participants practice using base ten blocks, place value mats and numeral cards for multiple-digit addition and subtraction. They will also learn how to do front-end estimation based on the base ten block model.
- In the fifth and last activity, participants view and discuss a video showing the difficulties children often have when asked to solve problems using procedures they don't understand and then showing children solving problems with understanding

Goals:

- To observe and learn how to conduct 1 on 1 performance assessments for the basic facts;
- To experience several constructivist mini-lessons for developing the addition/subtraction basic facts;
- To practice how to teach addition and subtraction fact strategies with Cuisenaire Rods;
- To become familiar with *Facts That Last* by Larry Leutzinger;
- To discuss the role of timed fact tests;
- To learn how to use base ten blocks to add and subtract multiple digit numbers
- To become comfortable with estimating sums and differences of multiple-digit numbers using front-end Estimation;
- To contrast the traditional base ten block approach for multiple -digit addition and subtraction with an approach that builds on children's natural problem solving ability.

Planning ahead for the Session:

Read the Materials. Read the following pages from the Staff Developer's Guide for this section.

View the tapes. Look at the video *Thinking with Numbers, Volume 2, and Video 1* by Kathy Richardson. Bridges Video includes students explaining subtraction strategies.

Gather the Materials. Each participant will need a set of 60 Unifix cubes, a set of story boards, counters, a tray of Cuisenaire Rods, base ten blocks (9 flats, 18 longs and 18 units per pair of participants), place value mats (one per pair of participants), 3 sets of numeral cards per pair of participants, The instructor will need a set of "Meaningful" flash cards, colored overhead pens,

and base ten blocks for the overhead. Check materials in Bridges Unit "Computational Procedures."

Practice Using the Manipulatives. Practice each example that you will show using the base ten blocks on the overhead. Space on the overhead platform is limited and you will need to be comfortable with the layout and procedures.

Prepare the Handouts and Overheads.

Handout 1 - The Hiding Assessment
Handout 2 - The Static Set cards
Handout 3 - Cognitively Guided Instruction Problem Types for Addition/Subtraction
Handout 4 - Addition Table
Handout 5a to 5l - Fantastic Facts for addition/subtraction
Handout 6 - Place value mat
Handout 7 - Addition/Subtraction Recording Sheet
Handout 8 - Helping Children Learn Mathematics

Make one copy of Handouts 1-7 for each participant.

Overhead 1a - The Hiding Assessment
Overhead 1b – Number Talk Recording
Overhead 2 - Cognitively Guided Instruction Problem Types for Addition/Subtraction
Overhead 3 – Counting on and back
Overhead 4 - Blank Addition/Subtraction Fact Table
Overhead 5 - Place value mat
Overhead 6 - Addition/Subtraction Recording Sheet
Overhead 7a-7b - Base ten blocks

Activity 1- Assessing student understanding of number

Activity Synopsis
The instructor illustrates the use individual assessments to determine the understanding of number combinations using the Hiding Assessment and to assess the understanding of the operations of addition and subtraction using the Static Set Assessment.

Materials:
Hiding Assessment
- Overhead 1-Hiding assessment recording sheet
- Handout 1-Hiding assessment recording sheet
- Unifix cubes

Static Set Assessment
- Handout 2-staic set equation cards cut apart
- Unifix cubes

Time:
15 minutes

Introduction to the Session: Describe the purpose of the assessments and introduce the hiding assessment and static set cards.

Say something like:

You will be seeing 2 different assessments that will help teachers to gather information on the child's understanding of addition and subtraction and number combinations. Similar to the previous assessment learned, the data collected from the assessments will be used to formulate appropriate instructional tasks for all students in the classroom.

Conducting the Activity

Assessment #1 - Hiding Assessment

Say something like:

Children who are able to subtract with ease and efficiency know the parts of numbers and see the relationship between composition and decomposition of numbers and addition and subtraction. They often explain how they got their answers by saying things like, "4 and 5 make 9, so 9 take away 5 is 4". Teachers report, however, that few children understand subtraction at this level. Instead of learning to decompose numbers, children often develop counting techniques for getting answers. Subtraction is difficult for these children because it is harder to get answers to subtraction problems using counters as a primary strategy than it is to get answers to addition problems. Even if the children memorize basic addition

facts, they often have difficulty applying what they know to subtraction if they learned these facts without recognizing the underlying relationship to the composition and decomposition of numbers. When children learn to subtract without decomposing numbers, they do not have the foundational skills necessary for solving problems with larger and more complex numbers. The key to subtraction is knowing the parts of numbers to ten so well that when given one part of a number, they automatically know the other part.

To begin,

Kathy Richardson's Hiding Assessment determines which number combinations the child knows by determining if they can tell the missing part of a number without having to figure it out. The interviewer will need 10 identical counters (unifix cubes) and Handout 1(student recording sheet).

Show the participants Overhead 1 of the Hiding Assessment and go through the assessment procedures that will need to be followed with the children. Call attention to the observations that need to made during the assessment**.**

Say something like:

This assessment is designed to identify instructional needs for children working at ability levels ranging from not knowing the parts of 3 to figuring out parts of numbers to knowing parts of numbers through 10 without figuring them out.

Show video of children performing the assessments. During marked discussion breaks on tape get participant feedback on what the child knows. If time allows have participants practice giving assessment with each other.

Assessment #2 - Static Sets Assessment

This task is developed to check to see how a child demonstrates a number operation using manipulatives. Many children will demonstrate the problem literally as opposed to operationally, thus creating a misconception of what an "operation"

Say something like:

Knowing addition and subtraction facts and understanding the operation can many times be two different things to a young child. This assessment highlights a misconception many students and teachers make in our rush for our children to know their math facts. This assessment was developed by Dale Phillips and is used widely in dap classrooms.

Prepare the three operation cards and 20-25 counters

Say something like:

Here I have a card with a number problem on it. Can you read this problem?
Here are some counters. Can you show me what this problem means using the beans.
Show the next two cards and repeat the same directions. Notice and record how the child responds to the different operations.

Say something like:

Think about the types of responses the child might show you or say.
What are the implications for how that child views addition and subtraction operations?
Why would some children make static sets?
How can this influence future mathematical ability in the classroom?

Possible responses:

The children's responses will range from actually making a graphic picture of the operation with the objects to correctly demonstrating all the equations, including the impossible card. The response is indicative of how that child has been taught and /or perceives number operations. Children who have memorized before they have a clear understanding of the operation typically demonstrate static sets .This is very evident when asked to do the impossible equation and the child has no problem demonstrating this.

Activity Summary for Assessing Student Understanding of Number
Say something like:

Should you ask the children how they get their answers?
What would be some grade level expectations of these assessments?
What type of classroom activities would need to be planned for the range of abilities in the classroom?
How can you show progress with these assessments?

Here are some possible responses:

- Having the child verbalize while working and after can help both the teacher and the child communicate their thinking. Children can sometimes correct a misconception they have had when they try to logically explain their thinking.
- During an assessment teachers do need to refrain from teaching or telling the answer. It is better to have lessons developed that create the need for children to develop these concepts. Competency develops over time and some children need activities that are presented in several different ways or over a period of weeks.
- Grade equivalence needs to be determined by correlating the goals of the assessment with district criteria.

- Checking for ongoing progress through daily observations can monitor progress. Ask occasional “What if “questions to children while they are working. This can quickly assess if a child is moving in the right direction.

Activity 2-A predominantly constructivist approach for teaching the addition and subtraction facts

Activity Synopsis
Participants experience the use of "meaningful flashcards" to promote understanding of number combinations and part-whole relationships.
Participants practice storytelling techniques using story boards to promote understanding of addition/subtraction operations.

Materials
Kathy Richardson video-Thinking with Numbers
Developing Number Concepts Book 2
Meaningful Flashcards
Storyboards/counters
Handout 3
Overhead 2

Time
30 minutes

Introduction to the Session: Describe the content and introduce the first activity

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In this session we will be exploring a constructivist approach in teaching addition and subtraction facts. Emphasis will be place on relationship between numbers and understanding of the operations.

Conducting the Activity

Meaningful Flashcards

Say something like:

Meaningful flashcards present children with various arrangements of quantities rather than symbols. When presenting them to children you are looking to determine if the child can recognize parts of a number and combine those parts without having to count all. The flashcards can be used along with number talks as part of a daily routine in the classroom.

Refer to Kathy Richardson Book 1 for BLM 20-27**.** Additional dot cards can be made using bingo stampers, stickers, toothpicks, etc. Present the participants with the cards and ask the following questions:

- ***How many?***
- ***How did you find out?***
- ***Who did it a different way?***

- ***How many would there be if I added one more?***
- ***Took one away?***

Story boards

Say something like**:**

Children using storyboards are common place in most primary classrooms. Unfortunately most of the addition and subtraction problems the children practice on the storyboards are joining and take away type problems. We will be practicing how to tell various types of problems using the Cognitively Guided Instruction Model.

To begin:

Pass out storyboards, counters and Cognitively Guided Instruction Problem Types Handout. Model a few of the problem and ask the group to partner up and practice telling each of the different models.

Say something like:

Which of the problems was the most difficult to tell? Easiest?
Why is it important to tell different level of addition and subtraction problems?
Can these problem types be used in other formats other than storyboards?

Activity 3 - A more behaviorist approach for teaching the addition/subtraction facts

Activity Synopsis
The instructor illustrates the following strategies for addition/subtraction: the use of the commutative property, number plus and minus one and two and zero, combinations of ten, plus and minus ten, plus and minus nine, doubles, doubles plus one, and doubles plus two. Participants practice using the Cuisenaire rods to illustrate each of the strategies.

Materials:
Overhead 4 - Blank Addition/Subtraction Fact Table; Handout 4 - Addition Table; colored overhead pens; Cuisenaire Rods; Literature – *Fish Eyes, Anno's Counting House, 1 Hunter;* Overhead 3- Counting on and back; Handout 5a to 5l "Fantastic Facts"; and *Facts That Last Addition and Subtraction* by Larry Leutzinger.

Time:
30 minutes

Introduction to the Session: Describe the content of this session and introduce the first activity.

We will be building the traditional addition/subtraction fact table by looking at strategies for learning the basic facts. Most traditional textbook series introduce the basic facts with counters such connecting cubes, then have students use the cubes to illustrate fact strategies such as counting on, counting back, counting up etc. and finally promote the memorization of the addition facts and then the subtraction facts. What we will see here is a variation on this traditional approach that uses Cuisenaire Rods. Many children grounded in the use of counters and counting strategies become stuck with counting. Counting is perfectly appropriate for a first grader, but not efficient for a fifth grader who should have the facts memorized. The use of the Cuisenaire Rods helps students make the bridge between counters and memorization because they are not scored and therefore not countable. The color association and the concrete aspect of the rods enables students who need the concrete to form a mental image of the strategies without relying on counting. We will also see samples from a commercial source for learning the facts, *Facts That Last* by Larry Leutzinger. Finally, we will discuss the role of timed fact tests.

To begin,

This process is adapted from an approach developed by Dr. Mahesh Sharma, Cambridge College, Boston MA. In order to use the rods, teachers and students have to become familiar with the color number association. This usually takes a class period or two for children, but longer for adults.

Conducting the Activity:

Introducing the Cuisenaire Rods

- Ask participants to take one rod of each color and build the staircase in the lid of their rod tray. Circulate making sure that the staircase contains ten different colored rods, placed in order, either from smallest to largest or largest to smallest. If rods are missing, ask participants if they know where e.g. the light green rod would go.
- Tell participants to keep the staircase intact and show different rods as they are requested.

Say something like:

If the white rod is one, show me the rod that corresponds to two. (red rod). Now, what rod would be ten? …five? …eight? …four? Etc.

- Check for understanding by having each participant show the requested rod. Repeat randomly for the harder rods for participants to remember. If need be, prompts can be suggested for some of the harder rods to remember: five gold rings; I ate (eight) a Tootsie Roll; four legs for Purple Barney; 7 black nights in a week; 3, 6, and 9 are the cool colors and the multiples of 3; and 2 (red) and 5 (yellow) are factors of 10 (orange). As preparation for what comes later, ask participants to show 14 the most efficient way (as the orange rod and the purple rod) or show 27 or 35 etc.

Using the Cuisenaire Rods for the fact strategies

- For plus and minus one, counting book literature like *Fish Eyes* by Lois Ehlert can be used. Starting with the staircase use a white rod to represent an additional fish added to random amounts of starting fish, e.g. opening the book to the page of 5 fish, have participants point to the five rod (yellow) on the staircase and place the additional white rod at the end of the yellow rod. Then ask what this shows visually. (That adding one to any number is equivalent to moving up one on the staircase.) In a similar manner subtracting one from five by covering the end of the yellow rod with the white rod results in a length equivalent to the four (purple rod).
- For plus and minus two, the same approach can be used with the red rod. This shows that adding or subtracting two skips one stair on the staircase. This builds the visual image for the mental strategy of skipping a number when adding or subtracting two. Oral repetition is important, as is the use of the commutative property. For example, in this case there are 6 equations that need illustration with the rods and repetition, e.g. 6+2=8, 2+6=8, 8=2+6, 8=6+2, 8-2=6, and 8-6=2. This is similar to a fact family, but goes beyond the fact family with the equations 8=2+6 and 8=6+2 that are important in developing the concept of equality. A common misconception with the equals sign is that children interpret it as the signal to give the answer and when asked to complete a missing addend equation like 3+•=6, they give the incorrect answer of 9.
- Adding and subtracting zero would only involve one rod.
- For the next strategy, combinations of ten, literature such as *Anno's Counting House* can be used. Have participants make the "Ten Sandwich" using orange rods as the starting and ending rods and finding all the two rod combinations that make the same length.

Say something like:

Say all the equations for ten. (0+10=10, 1+9=10, 2+8=10, 3+7=10, 4+6=10, 5+5=10, 6+4=10, 7+3=10, 8+2=10, 9+1=10, and 10+0=10) Pick up the complement for the purple rod (dark green rod). Say all six equations for what you have. (4+6=10, 6+4=10, 10=4+6, 10=6+4, 10-6=4, and 10-4=6). This process for learning the facts has two components, the rods that are not counted and oral repetition. In order to put something in long-term memory, the learner needs to hear the same fact repeated approximately 25 times. In the classroom, a teacher can randomly call on each child in the class to answer a question about the fact to be learned. Word problems should be included, e.g. If you have four stickers and you need ten, how many more do you need? Or, if you have seven stickers and your friend has three more than you do, how many does your friend have?

- The combinations of ten are crucial for facts where the sum is greater than ten and merit 2 or 3 class periods when teaching children. Children who count on their fingers to determine the harder addition facts don't know all the combinations of ten in order to build on those for the "make a ten" strategy which comes later.
- The next strategy that is taught is the doubles strategy which children naturally are drawn to. Ask participants to make a train using two 3 rods. With children we would ask them to estimate what one rod is the same size as their train (dark green or six rod). After the equation has been built with the rods, oral repetition helps put it in long term memory. For doubles in subtraction, ask questions like, "What two rods of the same color make eight? So 8-4=?
- The doubles plus one build on the doubles. For 4+5, ask participants to show the 4+5 train parallel to the 4+4 train. This easily shows that 4+5 is one more than 4+4.
- For ten plus a number or a number plus ten have participants show fourteen the quickest way and put it on the table as a train like it would be written (ten on the left and four on the right). In the classroom, teachers would continue with oral repetition of each equation shown. For a number minus ten, start with any teen number like 17 and ask participants to compare the train for that teen with the ten rod. Call their attention to the starting number and the ending number. Have participants describe what happens when they subtract ten, e.g. 17-10=7 (the answer is just the one's digit).
- Nine plus a number builds on ten plus a number. For 9+6, ask participants to make parallel trains for both 9+6 and 10+6 and line up the sixes. Ask, "Is 9+6 more than or less than 10+6? (Less) How much less? (one less). This is excellent for forming the visual image for the mental strategy that when you add 9 your sum is a teen where the ones digit is one less than the addend you started with. For a number minus nine, start with any teen number and ask participants to compare the train for the teen with the nine rod. Call their attention to the starting number and the ending number. Have participants describe what happens when they subtract nine, e.g. 17-9=8 (the answer is one more than the one's digit).
- After adding/subtracting ten and adding/subtracting nine, the doubles and near doubles can be revisited for the teen doubles and near doubles, like 7+7 and 7+8.
- Doubles plus two is another strategy that many children use, even without instruction in it. For example, for 5+7, ask participants to select the five rod and the seven rod

and then find the rod that is between five and seven (six). Have participants make two parallel trains, one for 5+7 and one for 6+6. This shows that for numbers that are two apart, the sum is double the middle number.

- After teaching all of the prior strategies, there are five remaining facts (6+3, 7+4, 8+3, 8+4, and 8+5) and their commutative pairs (3+6, 4+7, 3+8, 4+8, and 5+8). These are the rote facts, but it does help to compare the trains for these facts with the ten rod, e.g. 8+4 would be two longer than the ten rod. Show transparency 2 and refer participants to Handout 2 "Addition Table" which shows why these facts are not covered with the strategies mentioned earlier.
- Literature such as counting books like 1 Hunter by Pat Hutchins provide motivation for simple word problems and more complex ones, e.g. How many animals would there be if there were 3 giraffes and 6 crocodiles? Or how many animals are there in the whole book?

***Facts That Last* by Larry Leutzinger**

Say something like:

***Facts That Last: A Balanced Approach to Memorization* by Dr. Larry Leutzinger from the University of Northern Iowa is a series of four books - one for each operation. Each book contains 7 strategies to help students learn the basic facts, as well as activities, daily warm-ups, and daily practice for each strategy. Dr. Leutzinger has been providing these same activities, warm-ups, and daily practice to Iowa teachers for many years and they are now also available commercially.**

Show sample materials as time permits.

Timed Fact Tests

Say something like:

One of the most prevalent misunderstandings among some teachers and parents about recent reforms in mathematics is that children will no longer be required to learn their facts. This is far from true. The basic facts are the building blocks for all arithmetic activity - whether by paper/pencil, mental, or calculator methods. So the question is not whether they need to be learned, but how and when they are assessed. When we were in school, timed fact tests and competitions like "Around the World" were the norm. Most adults when asked what they remember from math when they were in school will mention timed fact tests. What is your opinion on timed fact tests? … Researchers have found that timed fact tests do not teach children facts or fact strategies. When children take repeated fact tests they become faster at using the facts and strategies that they already know. If they use the strategy of counting on their fingers, they become faster at counting on their fingers. If they are pressured with a time constraint while they are in the process of learning a mental strategy, they often revert back to a more familiar strategy such as using their fingers. Teachers need to ask themselves what their purpose is in giving children timed fact tests. Too often they become competitive exercises where

children compete with each other rather than with their prior results. If numeric fluency is the objective, than perhaps the variable that changes is the amount of time necessary to complete a fact test rather than the amount of correct answers in a constant time. Another consideration is that children should become faster at using strategies they have already internalized, but should not have a time pressure while they are learning a new strategy.

- Distribute handout 5a to 5l, "Fantastic Facts". Discuss how that the handout is different from commonly used fact tests, i.e. there is only one strategy per page, both addition and subtraction are included, and both horizontal and vertical formats are used.
- Display Overhead 3 and state that fact practice can be done with non-traditional formats like this one. With this format the instructor puts a number in the box and points to one of the numbers on one of the arrows to elicit a result of counting on or back 1, 2, or 3.

Activity Summary for Addition and Subtraction Facts

Say something like:

How does the behaviorist approach vary from the constructivist approach shown earlier?

Here are some possible responses.

- The behaviorist activities are teacher-centered. The teacher directs the students in what to do, does not provide choices or leveled activities for different needs. Oral repetition is prominent. Timed fact tests can be used.

Say something like:

Which process standards have been used in this section on the addition and subtraction facts? How?

Here are some possible responses.

- The Problem Solving Standard is used when children are introduced to the concepts of addition and subtraction through word problems on story boards.
- The Representation Standard is used when concrete materials represent individual facts
- The Communication Standard is used when learners write story problems to illustrate basic facts.
- The Connection Standard is used when basic facts are used to illustrate everyday applications.

Activity 4 - A more behaviorist approach for teaching the addition/subtraction algorithms

Activity Synopsis
The instructor illustrates how to use base ten blocks for multiple-digit addition and subtraction. Participants practice using base ten blocks to add and subtract with multiple digits. Finally, front-end estimation for addition is illustrated.

Materials:
Overhead 5 and Handout 6 - Place value mat; base ten blocks, numeral cards, one die for each pair of participants; base ten blocks for the overhead; Overhead 6 and Handout 7 - Addition/subtraction Recording Sheet; Overheads 7a-7b - Base ten blocks

Time:
15 minutes

Introduction to the Session: Describe the content of this session and introduce the first activity.

Base ten blocks have been used for many years to show multiple-digit addition and subtraction, but they are not a magic cure for the difficulties that many children experience. It is crucial that students be able to make the connections that adults naturally do between the use of the concrete materials and the step by step paper and pencil procedures, called algorithms. Because base ten blocks can also be used for decimals they are called flats, longs, and units instead of hundreds, tens, and ones. Base ten blocks can also be used to show why a newer technique for estimating than traditional rounding is effective. This method of estimation is often called "front-end" estimation.

To begin,

Each person will need a place value mat (Handout 6) showing columns for hundreds, tens and ones, 9 flats, 18 longs, and 18 units. For subtraction you will need numeral cards - 3 each for the numbers 0 to 9. A good way to introduce addition and subtraction with base ten blocks is by playing the games "Roll to 100" and "Roll to 0."

Addition

Conducting the Activity:

Using the base ten blocks for addition

- Show participants how to play "Roll to 100" using Overhead 5 and Handout 6 - Place value mat, overhead base ten blocks and a die. This game helps develop the readiness activity of trading that will be an integral part of the addition algorithm. Although the game ordinarily begins at zero, to shorten the game have participants start by showing

a number such as 67 on their place value mats with base ten blocks (6 longs and 7 units). Participants take turns rolling a die and adding that number of units to the ones column of their mats. If they have more than 10 units in that column, they trade ten units in for a long and place the additional long in the tens column. Play continues until one partner reaches (or passes) 100. The next step would be to record the results in another game of "Roll to 100" by keeping a running total after cubes are added. It is not necessary at this stage to show the renaming (what used to be called carrying), just the added cubes and the results should be shown. The final step in the game is to connect the work with the cubes to the symbolic recording

- For the actual addition algorithm, have children place the addend on the place value mat and the second addend below the mat e.g. 34+17 would have 3 longs and 4 units on the mat in their respective columns and 1 long and 7 units below the mat under their respective columns.
- Have the participants move the units from below the mat to the ones column on the place value mat. A ten-frame glued to the one's side of the mat is useful to decide if there are enough cubes for an additional long. If a new ten can be made, have the participants trade 10 units for a long and place it slightly above the others in the ten's column. Repeat the process in the ten's column. For larger numbers, if you are working with hundreds go on to the third column.
- Show additional examples such as 185+256 etc. as time permits. Have participants suggest 2 and 3-digit numbers to add.
- Show several examples using Overhead 6 and Handout 7 - Addition/subtraction recording sheet. Place close attention to how the extra place holders on the tens and ones templates are used for the renaming.

Say something like:

The hardest part of connecting the concrete to the paper and pencil algorithm or the record-as-you-go phase is getting children to remember to write down each step as they do it. The tendency is to put down the pencil and simply finish the task with the blocks. One suggestion is to have the children work in pairs, with one child in charge of the blocks and the other in charge of the recording. Children can then reverse roles with each new problem. To determine if children have adequately made the connection with the models, first see if they can do the manipulation and record the written steps. Then ask them to explain the written steps, including the little one at the top of the ten's column.

Subtraction

Say something like:

Multiple-digit subtraction is considerably harder than addition for students to do. Base ten blocks help make the procedure easier to understand, but don't expect children to have the same understanding that adults, including you will have.

Conducting the Activity:

Using the base ten blocks for subtraction

- Show participants how to play "Roll to 0". This game uses base ten blocks, place value mats, and dice to develop the readiness activity of trading for subtraction. Although the game ordinarily begins at 100, to shorten the game have participants start by showing the number 54 on their place value mats with base ten blocks (5 longs and 4 units). Participants take turns rolling a die and subtracting that number of units from the ones column of their mats. If they need to take away more than the number of units than they have in that column, they trade a long in for ten units and place them in the ones column. Play continues until one partner reaches (or passes) 0. The next step would be to record the results in another game of "Roll to 0" by keeping a running total after cubes are taken away. It is not necessary at this stage to show the renaming, just the subtracted cubes and the results. The final step in the game is to connect the work with the cubes to the symbolic recording.
- For the actual subtraction algorithm, have participants place the minuend on the place value mat and numeral cards for the subtrahend below the mat in the respective columns. The numeral cards serve as reminders for how much is to be removed from each column. For example, for 234 - 65 there would be 2 flats, 3 longs, and 4 units on the mat in their respective columns and a numeral card for 6 below the tens column and a numeral card for 5 below the ones column.
- Nothing should be removed until the full number of blocks can be taken off the mat. Begin with the one's column. To subtract 5 units, a long needs to be traded in for ten units and these moved to the ones column, resulting in 14 units in that column and 2 longs in the tens column. When trading from the ten's column to the one's column (one ten for ten ones) children should put the new units in the empty ten frame. This helps assure an accurate count. Then 5 units can be removed and placed on the numeral card for 5.
- To subtract 6 tens, a flat needs to be traded in for ten longs and these moved to the tens column, resulting in 12 tens. Then 6 tens can be removed and placed on the numeral card for 6 below the tens column. The answer is what remains on the mat, 1 flat, 6 longs, and 9 units or 169.
- Special attention needs to be given to include subtraction problems with zeros in the minuend, e.g. 400-159

Say something like:

The reason for the numeral cards is that the subtraction problem illustrates the take away action rather than comparison subtraction and the subtrahend (what is taken away) should not be shown with the blocks. By having the numbers subtracted on numeral cards under the mat, the subtraction problem can easily be checked by adding the number remaining on the mat to the number on the numeral cards below the mat.

- An interesting way to show subtraction from 100 with comparison subtraction is to use a flat as a mat for the 2-digit number to be subtracted. For example, for 100- 48 place 4 longs and 8 ones on top of the flat representing 100. The difference or answer to the subtraction

problem is the part of the flat not covered by the 4 longs and 8 ones. In this case, the difference would be 2 ones and 5 longs or 52. Four different computational exercises can be described as 100-48=52, 100-52=48, 48+52=100, and 52+48=100.

- Show several examples using Overhead 6, highlighting how the place holder boxes are used for subtraction for the renaming.

Front-End Estimation

Say something like:

Most adults did not spend much time with estimation when they were in school and if they did, they probably just used rounding. As we have seen earlier, estimation and mental computation are quite often used as often if not more than paper and pencil computation in everyday applications. Researchers interviewed thousands of adults and children who were good estimators about the ways they estimated. With addition they found that many were actually adding the largest place values first and then adjusting according to the place values that followed. This technique has been named "front-end" estimation.

Conducting the Activity:

Using the base ten blocks to illustrate our natural inclination to deal with the largest place values first

- Tell participants that you are going to put a random collection of overhead base ten blocks on the overhead and give them 3 seconds to estimate the quantity shown. Turn off the overhead and place a handful of base ten blocks on the overhead. Be sure that you have a few hundreds. Overheads 7a-7b can be used for smaller versions of base ten blocks. After three seconds, turn off the overhead and ask participants to jot down their estimate on a sheet of scratch paper. Have them share with a partner and then call on several to give their estimates and an explanation for how they arrived at an estimate. Most of us will start with counting the number of hundreds first and then move to the tens next and then ones as time permits.

Say something like:

Most of us will count the number of largest pieces first and to do so is a child's natural inclination when using the base ten blocks or unifix cubes when adding and subtracting with multiple-digits. This also illustrates the usefulness of "Front-End" estimation. When efficient estimators were interviewed the interviewers discovered that many had personally invented this substitute procedure for rounding when adding a column of multiple digit numbers.

- Have 8 to 10 participants give random prices that might appear on a grocery store tape. Write them in a column on a blank overhead and have participants estimate the

total bill by rounding. Have one or two describe their result and the process they used.

- Illustrate front-end estimation by adding the dollar amounts first and then adjusting this estimate by determining how many more dollars can be formed with amounts from the dime's column.

Ask something like:

Although many of us are not as comfortable with front-end as with traditional rounding, can you see any advantages to practicing so that you are comfortable?

Here is a possible response.

- With "Front-End" what you see is what you add. With rounding, if you round up, you are adding numbers that you don't see.

Activity 5 –A predominately constructivist approach for teaching the addition and subtraction algorithms

Activity Synopsis
Participant will view and discuss a video showing the difficulties children often have when asked to solve problems using procedures they don't understand and then showing children solving problems with understanding**.**

Materials
Kathy Richardson video: Thinking with Number Vol. 2

Time
35 minutes

Introduction to the Session: Describe the content and introduce the activity

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In this video you will see children solving computation problems in a way that is meaningful to them, rather than following a procedure.

Conducting the Activity

View the video. At different stopping points
Say something like:

Did you notice that not one of the children said, "I can't do this problem."
What does the environment of the classroom have to do with the teaching?
What would you want to change/add to the lesson?
How are children's different levels of understanding being met?

Activity Summary for Addition and Subtraction Algorithms

Say something like:

How does the behaviorist approach vary from the constructivist approach shown earlier?

Here are some possible responses.

- The behaviorist activities are teacher-centered. The teacher directs the students in what to do, does not provide choices or leveled activities for different needs. Each child is asked to do the same exercises at the pace of the class. Beginning with the ones column runs counter to the child's natural inclination to begin with the largest blocks.

Say something like:

> **Which process standards have been used in this section on the addition and subtraction algorithms? How?**

Here are some possible responses.

- The Problem Solving Standard is used when children are introduced to the concepts of addition and subtraction with multiple-digit numbers through word problems.
- The Representation Standard is used when concrete materials represent individual addition/subtraction problems.
- The Connection Standard is used when algorithms are used to illustrate everyday applications.
- The Communication Standard is used when participants explain their thinking to their partners and the whole group.